



# Special afternoon on Diffuse Interface Models and Related Problems

*Dipartimento di Matematica "F. Casorati" e IMATI-CNR, Pavia — February 7, 2017*

*Organized by: P. Colli (Unipv), E. Rocca (Unipv), L. Spinolo (Imati) – registration at [elisabetta.rocca@unipv.it](mailto:elisabetta.rocca@unipv.it)*

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15:00-15:45

*What is the shape of an object flowing in a fluid that has the least drag ?*

**Key-Fong Lam** (University of Regensburg, Germany)

*In shape optimisation problems, the goal is to find an optimal shape that achieves some desired properties. One of the most studied shape optimisation problem is that of drag minimisation, that is, what is the shape of an object flowing in a fluid that has the least drag forces. This has important applications and consequences in the design of ships, racecars, wind turbines and airplane wings. In this talk, we present a mathematical formulation of the drag minimisation problem in a stationary Navier-Stokes flow. The formulation utilises a phase field approach, and we can prove the existence of at least one optimal shape. Furthermore, we can also derive necessary optimality conditions rigorously, and together with a gradient flow approach, we use these conditions to numerically solve for the optimal shapes of drag minimisation.*

15:45-16:30

*Quasi-static problem for the Cahn--Hilliard equation on the boundary*

**Takeshi Fukao** (Kyoto University of Education, Japan)

*The well-posedness for a system of partial differential equations and dynamic boundary conditions is discussed. This system is a sort of transmission problem between the dynamics in the bulk and on the boundary. The Poisson equation for the chemical potential, the Allen--Cahn equation for the order parameter in the bulk are considered as auxiliary conditions for solving the Cahn--Hilliard equation on the boundary. Recently the well-posedness for the equation and dynamic boundary condition, both of Cahn--Hilliard type, was discussed. Based on this result, the existence of the weak solution and its continuous dependence on the data are proved. This study is based on the recent joint works with P. Colli (Pavia, Italy).*

16:30-17:00 *Coffee-break*

17:00-17:45

*Well-posedness of a diffuse-interface model for two-phase incompressible flows with thermo-induced Marangoni effect*

**Hao Wu** (Fudan University, Shanghai, China)

*We consider a diffuse-interface model that describes the dynamics of two-phase incompressible flows with thermo-induced Marangoni effect. The surface tension, fluid viscosity and thermal diffusivity are allowed to depend on the (relative) temperature. We establish the existence and uniqueness of local strong solutions when the spatial dimension is two and three. Then in the two-dimensional case, assuming that the initial temperature is suitably bounded with respect to the coefficients of the system, we prove the existence of global weak solutions as well as the existence and uniqueness of global strong solutions.*